LOCAL POSITIONING SYSTEM (LPS)
FOR A RADIO-WAVE-CONTROLLED FLYING ROBOT

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ABSTRACT

Mobile robots are classified into 3 main groups; on-ground wheeled or legged robots, in-water swimming robots, and in-air flying robots. Movement of wheeled and legged robots is limited very much by feature of the working area, while degree of freedom of movement by flying robot is extent to 3 dimension. Flying robot in the future is expected to be another solution for agricultural purposes, resources exploration, communication and transportation. In this research, radio-wave-controlled flying robot is selected as our platform due to its capacity to hover and good flexibility. To automate the robot to fly freely in space, information about the robot’s current position is necessary. Local Positioning System (LPS) is proposed in this paper for positioning the robot in a local area. LPS indicates the current position in 3-dimensional space of the flying robot by modifying the concept of spheres intersection in Global Positioning System (GPS). Synchronization of transmitter and receivers clocks is unnecessary since absolute travelling is not used in position calculation. LPS is expected for local purpose, therefore, ultrasonic is used as a carrier for determining time differences in stead of electro-magnetic wave used in GPS. In LPS, one ultrasonic transmitter is equipped on the flying robot while four receivers are on ground bases. Four spheres intersect at only a point where the position of the flying robot in the space is. Microprocessor is programmed to calculate the current position of the robot. When ultrasonic pulse from the transmitter arrives to all the four receivers, time differences between pairs of receivers can be determined and processed to calculate the position. The output is
then shown as a \( x \), \( y \), and \( z \) coordinate relative with reference point in space. Two experiments are conducted and tested in the research to investigate maximum working space and accuracy of position determined by LPS.

**Keywords:** Local Positioning System, Ultrasonic, and Radio-Wave-Controlled Flying Robot

### 1. INTRODUCTION

By consideration of the working environment, mobile robots are classified into 3 main groups; on-ground wheeled or legged robots, in-water swimming robots, and in-air flying robots. Movement of wheeled and legged robots is limited very much by feature of the working area, while degree of freedom of movement by flying robot is extent to 3 dimension. The flying robot could be applied in better, cheaper, faster and even more precise applications. For example, in aerial mapping application if an airplane is used, it costly consumes a lot of fuel. An experienced pilot has to fly efficiently with less overlapped areas. Unlike the real airplane, automatic-controlled flying robot makes efficient cost. Less fuel and operation time can be expected because of the capability to fly following trajectory planned in advance. Flying robot in the future is expected to be another solution for agricultural purposes, resources exploration, communication and transportation.

There are some significant researches on flying robot. An autonomous helicopter using information from CDGPS and computer vision in decision making is developed at Aerospace laboratory Stanford university [1], [2]. Information from multiple CDGPS, 2 GPS with 4 antenna on board and 1 GPS with 1 antenna on ground, is used to calculate both current position of helicopter and its heading direction. For computer vision system, CCD camera is applied to recognize a moving object on ground for the tracking purpose. However, the object image is processed on ground, the result of recognition is sent to the helicopter later. Since control of position and heading direction is conducted on board where only simple microprocessor is equipped, complicated control algorithm is impossible. Only proportional control algorithm is applied. In the case that signal from satellites disappears by some reasons, the
helicopter will not able to recognize its position and direction. This problem is a big issue to be solved. GPS Pseudolite is introduced to solve the problem of disappearance of signal from satellite [3]-[6]. GPS Pseudolite is installed on ground to generate a pseudo signal similar to signal transmitted from satellite to receivers. In normal operation, a flying robot receives signal from satellites for positioning. When signal from satellites disappear, pseudo signal from GPS Pseudolite is used in stead. Since signal from all satellites including GPS Pseudolite are related each other, calibration of transmitted signals is necessary and important. This drawback can be solved if the system is set locally not globally and if only a single signal is transmitted in stead of multiple signals from multiple satellites.

To avoid difficulty in developing flying robot, radio-wave-controlled flying robot is selected as our platform due to its capacity to hover and good flexibility. In our flying robot, all the commands to control the robot movement are sent from radio-wave controller, in the other word, the robot’s brain is on ground. A microprocessor is equipped on ground, it interfaces with the radio-wave controller to control the robot. By this architecture, complicated control algorithm is possible since there is no constraint about weight, dimension, and others on ground. In radio-wave-controlled flying robot, one control pulse from the radio wave controller is used to control one servomotor. With deterministic series of pulses, all servomotors on the flying robot are controlled as desired. The controlling pulses are generally modulated before transmission. The well-known modulations techniques used in recent radio-wave-controlled flying robot are AM (Amplitude Modulation), FM (Frequency Modulation) and PCM (Pulse Code Modulation). The radio wave receiver, after receiving the signal, demodulates the signals and then sends command to each servomotor to control movement of the flying robot.

To automate the robot to fly freely in the space, generally, the following information is required.
1. Information of the robot’s current position as coordinate of $x$, $y$, and $z$ in 3-dimensional space
2. Information of the robot’s current orientation of row, pitch, and yaw angles
3. Information of the robot’s current altitude
4. Information of external world, etc.
In this research, a sensor used to detect information of robot’s current local position, named Local Positioning System or LPS, is proposed and developed. LPS indicates the current position in 3-dimensional space of the flying robot by modifying the concept of spheres intersection in Global Positioning System (GPS). LPS is expected for local purpose, therefore, ultrasonic is used as a carrier for determining time differences in stead of electro-magnetic wave used in GPS. In LPS, one ultrasonic transmitter is equipped on the flying robot while four receivers are on ground bases. Four spheres intersect at only a point where the position of the flying robot in the space is. Microprocessor is programmed to calculate the current position of the robot. When ultrasonic pulse from the transmitter arrives to all the four receivers, time differences between pairs of receivers can be determined and processed to calculate the position. There are many merits of using LPS. Since LPS does not rely on external satellites, the problem of signal disappearance found in GPS is solved by increasing number of receivers on ground easily which is a hard implementation in the case of GPS. This practice also helps to improve system accuracy at the same time. Calibration of transmitted signal is unnecessary in LPS since there is only one transmitter which is equipped on the flying robot.

2. POSITIONING SYSTEM IN FLYING ROBOT

Since Local Positioning System (LPS) is modified for local purpose from Global Positioning System (GPS), the concept of positioning by GPS is mentioned first.

2.1 Global Positioning System (GPS)

The Global Positioning System (GPS) is a worldwide radio-navigation system formed from a constellation of 24 satellites and their ground stations. GPS uses these “man-made stars” as reference points to calculate positions accurately to a matter of meters. In fact, with advanced forms of GPS measurements to better than a centimeter is possible. In a sense, it’s like giving every square meter on the planet a unique address [7]. GPS receivers have been miniaturized to just a few integrated circuits and so are becoming very economical. That makes the technology accessible to virtually everyone.
These days GPS is finding its way into cars, boats, planes, construction equipment, farm machinery, or even laptop computers [8].

Ground Stations also known as the “Control Segments” monitor the GPS satellites, checking both their operational health and their exact positions in space. The master ground station transmits corrections for the satellites’ ephemeris constants and clock offsets back to the satellites. The satellites can then incorporate these updates in the signals they send to GPS receivers. There are five monitor stations located at Hawaii, Ascension Island, Diego Garcia, Kwajalein, and Colorado Springs.

Spheres intersection is the fundamental concept in determining global position. Suppose distance from a place on earth to satellite is $d_1$. Knowing this distance of $d_1$ from a particular satellite narrows down all the possible locations of the unknown place in the whole universe to the surface of a sphere that is centered on this satellite and has a radius of $d_1$ as shown in figure 1.

Next, if distance to a second satellite is found at $d_2$ away. The unknown place now is not only on the first sphere but also on a sphere that is $d_2$ from the second satellite. In the other word, this unknown place is somewhere on the circle where these two spheres intersect as shown in figure 2.

If a third measurement from another satellite is found at $d_3$, that narrows the position down even further, to the two points where the $d_3$ sphere cuts through the circle that is the intersection of the first two spheres. So by ranging from three satellites position of the unknown place is narrowed to just two points in space. To decide which one is true location a fourth measurement is the solution. But usually, one of the two points is a ridiculous answer either too far from Earth or moving at an impossible velocity and can be rejected without a measurement.

Distance to a satellite is determined by measuring how long a radio signal takes to reach an unknown place from that satellite. To make the measurement, both the satellite and receiver of the unknown place have to generate the same pseudo-random codes at exactly the same time. By comparing how late the satellite’s pseudo-random code appears compared to the receiver’s code, traveling time can be determined. Then, just multiply that travel time by the speed of light, the distance is obtained.
2.2 Local Positioning System (LPS)

Comparison between GPS and LPS is stated in table 1.

GPS uses 3 or more satellites as fixed bases to transmit radio wave signal. A receiver is equipped on a vehicle to find absolute position in global area by mean of absolute travelling time. Absolute position information is obtained at the vehicle. While LPS has a transmitter equipped on flying robot to transmit ultrasonic signal. Four receivers are fixed at specific places on ground. By measuring travelling time differences of ultrasonic to the receivers, local position can be determined at the ground bases. These ground bases are connected with microprocessor to calculate the robot’s current position. Synchronization between transmitter and receivers clocks is unnecessary in LPS. The microprocessor can be also used to control motion of flying robot directly or indirectly. Programming flowchart for determination of position by LPS is shown in figure 3.

Ultrasonic pulses are generated in LPS by ultrasonic transmitter. Ultrasonic receivers at ground bases receive pulses at different times depending on distances between transmitter and receivers. The time differences detected at the receivers are the input to microprocessor which are programmed to calculate a coordinate of \( x \), \( y \), and \( z \) in local position. The result of calculation is displayed as the robot’s current position via LCD. Photo in figure 4 depicts the flying robot with equipment of LPS system.

2.2.1 Position Calculation in LPS

By the concept of spheres intersection, 4 spheres centered at each receiver intersect at only a point where the position of the transmitter and also the flying robot is. The arrangement of receivers is shown in figure 5. In LPS, synchronization between transmitter and receivers clocks is unnecessary since absolute travelling time is not used in position calculation.

Equations for calculating transmitter position follow:

\[
x^2 + y^2 + z^2 = r_1^2 = (t_1 * s)^2
\]

\[
(x-a)^2 + y^2 + z^2 = r_2^2 = (t_2 * s)^2 = ((t_1 + \Delta t_2) * s)^2
\]
\[ x^2 + (y-b)^2 + z^2 = r^2 = (t_3^*s)^2 = ((t_1 + \Delta t_3)^*s)^2 \] (3)

\[ (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2 = (t_4^*s)^2 = ((t_1 + \Delta t_4)^*s)^2 \] (4)

where \( t_i = \) travelling time from transmitter to receiver no \( i \)

\[ \Delta t_i = t_i - t_1 = \text{time differences between receiver no 1 and receiver no } i, \text{ when } i \neq 1 \]

\( s = \) velocity of sound in air (\( = 341 \text{ m/s at room temperature} \))

\( a, b, \) and \( c = \) distances of receivers’ arrangement

\( x, y, \) and \( z = \) current position of flying robot relative to the position of receiver no 1.

From equations (1)-(4), there are 4 unknown variables; \( x, y, z, \) and \( t_1 \). By solving for the solution of these four equations,

\[
y = \begin{cases} 
-\frac{J - \sqrt{J^2 - 4IK}}{2I} & \text{when } \Delta t_3 \text{ is negative} \\
\frac{J + \sqrt{J^2 - 4IK}}{2I} & \text{otherwise}
\end{cases} 
\] (5)

\[
x = \frac{2b\Delta t_3 y + s^2 \Delta t_2 \Delta t_4^2 - b^2 \Delta t_2}{2a\Delta t_1} - s^2 \Delta t_2^2 \frac{a}{2} + \frac{a}{2}
\] (6)

\[
z = \frac{2b\Delta t_4 y - b^2 \Delta t_4}{2c\Delta t_3} + \frac{s^2 \Delta t_3 \Delta t_4}{2a} + \frac{a^2 + b^2 - s^2 \Delta t_4^2 - 2ax - 2by}{2c} + \frac{c}{2}
\] (7)

where,

\[
I = 1 - \frac{b^2}{s^2 \Delta t_3^2} + \frac{b^2}{a^2 \Delta t_3^2} + \frac{b^2 \Delta t_2^2}{c^2 \Delta t_3^2} + \frac{b^2 \Delta t_2^2 \Delta t_3^2}{c^2 \Delta t_3^2} + \frac{2b^2 (\Delta t_2 - \Delta t_4)^2}{c^2 \Delta t_3^2}
\] (8)

\[
J = \frac{bs^2 \Delta t_2^2}{a^2} + \frac{2bs^2 \Delta t_4^4 - 2bs^2 \Delta t_2 \Delta t_4 - bs^2 \Delta t_3 \Delta t_4 + bs^2 \Delta t_3 \Delta t_3 - b^3}{c^2} + \frac{b^2 \Delta t_4}{\Delta t_3} - \frac{b^3 (\Delta t_2 - \Delta t_4)^2}{c^2 \Delta t_3^2} + \frac{b^3}{s^2 \Delta t_3^2}
\]

\[
+ \frac{bs^2 \Delta t_2^2 \Delta t_2 - bs^2 \Delta t_2^2 \Delta t_4 - bs^2 \Delta t_3^3 + bs^2 \Delta t_3^3 - 2b^3 (\Delta t_2 - \Delta t_4)^2}{c^2 \Delta t_3^2} - \frac{b^2 \Delta t_2^3}{a^2 \Delta t_3} - \frac{b^3 \Delta t_2^3}{a^2 \Delta t_3^2} - 2b \]

\[
K = \frac{4b^2}{4} + \frac{a^2 + c^2 + 2s^2 \Delta t_3 \Delta t_4 - s^2 \Delta t_3^2 - 2s^2 \Delta t_4^2}{4} - \frac{b^2 \Delta t_4}{2\Delta t_3} + \frac{b^4 \Delta t_2^6 (\Delta t_2 - \Delta t_4)^2}{4c^2 \Delta t_3^3} + \frac{b^4 \Delta t_2^2}{4a^2 \Delta t_3^2} + \frac{b^2 \Delta t_2^2 \Delta t_3^3}{2a^2 \Delta t_3}
\]

\[
+ \frac{b^4 + 2s^4 \Delta t_2 \Delta t_3 \Delta t_4 (\Delta t_2 - \Delta t_3 + \Delta t_4) - 2s^4 \Delta t_2^3 \Delta t_3^2 - 2s^4 \Delta t_2 \Delta t_4^3 - s^4 \Delta t_2^3 \Delta t_4^2 + s^4 \Delta t_3^2 \Delta t_4^2}{4c^2}
\]

7
\[ + \frac{s^4 \Delta t_2^4 + s^4 \Delta t_4^4 + 4b^2 s^2 \Delta t_4^2 (\Delta t_2 - \Delta t_4) - 2b^2 s^2 \Delta t_3 (\Delta t_2 - \Delta t_4)}{4c^2} + \frac{s^2 \Delta t_2^2 (\Delta t_2 - \Delta t_3)^2 - 2b^2 s^2 \Delta t_2^2}{4a^2} \]

\[ + \frac{b^2 s^2 \Delta t_2^3 + b^2 s^2 \Delta t_4^3 - b^2 s^2 \Delta t_3 \Delta t_4 - b^2 s^2 \Delta t_2 \Delta t_4^2 + b^4 (\Delta t_2 - \Delta t_4)}{2c^2 \Delta t_3} - \frac{b^4}{4s^2 \Delta t_3^2} \]

(10)

2.2.2 LPS Transmitter Circuit

Ultrasonic is the sound wave whose frequency is above the audible range of 20 kHz (audible range is from 20 Hz to 20 kHz). In this research, ultrasonic of the frequency of 40 kHz is used for transmission. Block diagram of LPS transmitter circuit is shown in figure 6.

Astable circuit generates an adjustable frequency controlled by a specific resistor. The generated frequency is then divided by \(2^x\) where \(x\) is number of reduction stage. In the prototype circuit, 10 stages of reduction are used. At every falling edge, monostable circuit is trigged to give a pulse with 0.2 ms pulse-width. This pulse is used to turn on a switch gate so that the other astable signal, whose frequency is at 40 kHz, is delivered for amplification. The signal after being amplified is finally used to drive the head of ultrasonic transmitter (Tx).

An astable circuit is used to generate high frequency. This high frequency is then reduced to the lower required frequency by a counter. Instead of a frequency signal, a monostable circuit is then used to generate a pulse signal to turn on the switch gate, which connects carrier frequency signal from another astable circuit with an amplifier before ultrasonic transmitter. Finally, the amplified signals whose frequency is 40 kHz with 0.2 ms envelope pulse-width is sent to the ultrasonic transmitters in anti-phase to obtain the maximum output. The ultrasonic pulse is transmitted every more or less 1 s by adjusting the transmitter circuit so that microprocessor can finish calculating a position before the next coming of successive pulse.
2.2.3 LPS Receiver Circuit

Block diagram of LPS receiver circuit is shown in figure 7. The receiver firstly receives ultrasonic signal in a pulse envelope sent from the transmitter. The signal is then amplified. It then passes a low pass filter to eliminate 40 kHz carrier frequency signal and noise out. Since the positioning program uses logic 0 to represent the arrival of pulse, inverter is equipped next. The pulse is finally adjusted to the voltage that is compatible with TTL level of 5 volts to be connected with input port of microprocessor.

2.2.4 LPS Positioning Program

The positioning program is written to do polling check four input ports which are connected with four receiver circuits. Microprocessor free-running counter runs all the time. The current value from this counter is loaded to memory whenever ultrasonic pulse arrives to each receiver. Polling check is done continuously until all the receivers receive ultrasonic pulses. Subtraction between counter values returns time differences between pairs of receivers. Three time differences are used to solve for three unknown variables of coordinate of flying robot in space. The output is then displayed at LCD.

Figure 8 shows the flowchart of LPS polling check. LPS firstly checks whether each receiver has received pulse already or not. If pulse is detected, time marker of each receiver is marked and the current counter number is loaded into the receiver’s time variable. Polling check runs until the time markers are all marked, meaning that all receivers have received ultrasonic pulses. The system is reset if a receiver does not receive pulse within a reasonable period.

Position calculation is the most important part of LPS positioning program since coordinate of flying robot in space will be correct or not depending mostly on this part. After the polling check successfully completes, time differences between pairs of receivers are then found. These time differences are the required parameters for substitution into the positioning equations, 4 equations of sphere centered at the receivers and intersect at the transmitter. LPS computes the position according
to equations rewritten compatibly with microprocessor programming. Figure 9 shows time differences between receivers.

The result of calculation in hexadecimal is converted to decimal firstly before being displayed via LCD.

3. EXPERIMENTS AND RESULTS

Transmitter model of 400ST160 and receiver model of 400SR160 of air ultrasonic ceramic transducer pair, dimension shown in figure 10, are used in the experiment. Specification of the ultrasonic transducers is shown in Table 2.

Two experiments are conducted and tested in the research.
1. Experiment to investigate maximum working space by the developed LPS
2. Experiment to investigate accuracy of position determined by the developed LPS

In the first experiment, maximum range that ultrasonic pulse is still detectable at the receiver in the developed circuit is about 8 m. By proper arrangement of four ultrasonic receivers, the maximum working space of the sphere with the radius of 8 m is obtained. Here, all the receivers are located at four distinct locations close to the center of upper semi-sphere. In general, the detectable distance depends directly on the specification of ultrasonic transmitter and ultrasonic receiver. With higher power ultrasonic heads, longer detectable range can be expected.

In the second experiment, positions of 100 different locations in the working space are tested by LPS. The experimental arrangement is set by locating 4 receivers at the position; (0, 0, 0), (3, 0, 0), (0, 3, 0), and (3, 3, 1) respect to receiver number 1 and the coordinates are in meter unit. Average position error in three directions is shown in table 3. The causes of error result from error in locating four ultrasonic receivers, error from constant sound velocity at 341 m/s used in the formulas, and error from resolution of microprocessor used to calculate distances, etc. The reason that error in y direction is smaller errors in x and z directions is that in the microprocessor programming, position in y direction is derived first. Position in x direction is derived from y direction and position in z direction is derived
from both y and x directions respectively. Thus, error is accumulated. The error can be reduced by calculation of position in each direction separately.

One weak point of LPS is that ultrasonic pulse is blocked or disturbed easily by external obstacles. Incorrect position or, even worse, no position might be obtained.

4. CONCLUSION

The final objective of this research is to automatically control flying robot. In order to do so, the information of current position of the robot is necessary. Since accuracy of standard GPS is not good and high accuracy GPS is expensive, thus cheaper positioning sensor for local purpose is required. LPS was developed by modifying the concept of spheres intersection seen in GPS to sense the robot’s position in a specific area with acceptable accuracy. In LPS, one ultrasonic transmitter is equipped on the flying robot while four receivers are on ground bases. Four spheres intersect at only a point where the position of the flying robot is in the space. Synchronization of transmitter and receivers clocks is unnecessary since absolute travelling is not used in position calculation. From the experiments, the average error of LPS was found about 15% in 3 dimensional space while the maximum working area was represented by a sphere with 8 m in radius. There are many merits of using LPS. Since LPS does not rely on external satellites, the problem of signal disappearance found in GPS is solved by increasing number of receivers on ground easily which is a hard implementation in the case of GPS. Calibration of signal is also not a problem in LPS since there is only a single signal transmitted instead of multiple signals used regularly in GPS.

REFERENCES


FIGURES

Figure 1 A Sphere

Figure 2 Two Spheres Intersection
Generating ultrasonic pulses by transmitter

Ultrasonic pulses

Receiving ultrasonic pulses by receivers

$\Delta t_i$

Calculating current position of robot by microprocessor

Current position

Displaying position of $x$, $y$, and $z$ via LCD

Programming of positioning equations

Figure 3 Programming Flowchart of LPS

Figure 4 Flying Robot with LPS
Figure 5 Receivers’ Arrangement (a) Top View (b) Front View

Figure 6 Block Diagram of LPS Transmitter Circuit
Figure 7 Block Diagram of LPS Receiver Circuit

Figure 8 Flowchart of Polling Check
Transmitter circuit

Rx

Receiver circuit#1

Rx

Receiver circuit#2

Rx

Receiver circuit#3

Rx

Receiver circuit#4

8255 port

Microprocessor (MC68HC11A1)

LCD

Figure 9 Time Differences between Receivers

Tx 40 kHz (pulse width ≈ 0.2 ms)
Figure 10 Dimension of Ultrasonic Transducer

<table>
<thead>
<tr>
<th></th>
<th>GPS</th>
<th>LPS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transmitter</strong></td>
<td>On Satellites</td>
<td>On Flying Robot</td>
</tr>
<tr>
<td><strong>Receiver</strong></td>
<td>On Vehicle</td>
<td>On Ground Bases</td>
</tr>
<tr>
<td><strong>Medium</strong></td>
<td>Radio Wave</td>
<td>Ultrasonic (Sound Wave)</td>
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<td><strong>Area</strong></td>
<td>Global</td>
<td>Local</td>
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<tr>
<td><strong>Algorithm</strong></td>
<td>Absolute Time Checking</td>
<td>Travelling Time Difference</td>
</tr>
</tbody>
</table>

Table 1 Comparison of GPS and LPS
**400ST160** Transmitter

**400SR160** Receiver

**Center Frequency** 40.0±1.0 kHz

**Bandwidth** (-6dB) 400ST160 2.0 kHz

**Bandwidth** (-6dB) 400SR160 2.5 kHz

**Transmitting Sound Pressure Level** 120 dB min

at 40.0 kHz; 0 dB re 0.0002 µbar

per 10 Vrms at 30 cm

**Receiving Sensitivity** -65 dB min

at 40.0 kHz; 0 dB = 1 volt/µbar

**Capacitance** at 1 kHz ±20% 2400 pF

**Max Driving Voltage** (cont.) 20 Vrms

**Total Beam Angle** –6dB 55° typical

**Operation and Storage Temperature** -40 to 85°C

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**Table 2 Specification of Ultrasonic Transducers Used in the Experiment**

<table>
<thead>
<tr>
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<th>Average Error (%)</th>
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<tbody>
<tr>
<td>x</td>
<td>7.38</td>
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<tr>
<td>y</td>
<td>4.56</td>
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<tr>
<td>z</td>
<td>33.50</td>
</tr>
</tbody>
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**Table 3 Average Position Error in Three Directions**

20